# Learning without Missing-At-Random Prior Propensity-A Generative Approach for Recommender Systems

Yuanbo Xu, Fuzhen Zhuang, En Wang†, Chaozhuo Li, and Jie Wu, *IEEE Fellow*

**Abstract**—A common assumption in the literature is that the missing ratings are missing at random (MAR), meaning that the probability of observing a rating does not depend on its value. However, this assumption is often violated in real-world scenarios, where users tend to provide ratings for items they like or dislike more than average, leading to a missing not at random (MNAR) situation. To address this problem, some researchers have proposed to use explicit MAR feedbacks to estimate the propensities of unobserved implicit MNAR feedbacks. However, collecting explicit MAR feedbacks is costly and time-consuming and may not reflect users' true preferences. Moreover, most of these methods have only been tested on synthetic or small-scale datasets, and their applicability and effectiveness in real-world settings without MAR feedbacks remain unclear. To this end, we aim to predict MNAR ratings without MAR prior propensities by exploring the consistency between MAR and MNAR feedbacks and bridging the gap between them. From the empirical study and preliminary experiment, we hypothesize that *user preferences* can be treated as the common prior propensity for both MAR and MNAR generative processes. In this way, we extend this hypothesis to a more general MNAR scenario: user preferences learned from MNAR can partially substitute for the prior propensities derived from MAR feedbacks for MNAR recommendation tasks. To validate our hypothesis and approach, we develop a lightweight iterative probabilistic matrix factorization framework (lightIPMF) as a practical method of our methodology, utilizing user preferences extracted from MNAR, not MAR, to estimate MNAR feedbacks. Finally, the experimental results show that modeling user preferences can effectively improve MNAR feedback estimation without MAR feedback, and our proposed lightIPMF outperforms the state-of-the-art MNAR methods in predicting MNAR feedbacks.

**Index Terms**—Missing-Not-At-Random, Generative Model, Recommender Systems

## 1 Introduction

Recommender systems are widely emerging in various domains to provide personalized suggestions to users based on their preferences and behaviors [1, 2]. However, a major challenge in building effective recommender systems is coping with the sparsity and incompleteness of the user-item feedback data, where most ratings are missing. A common simplifying assumption in existing recommendation models is that the ratings are missing at random (MAR), meaning that the probability of observing a rating does not depend on its value or other factors. However, this assumption is often unrealistic and inaccurate in real-world scenarios, where users tend to provide ratings for items that they have strong opinions about, either positive or negative, leading to a missing not at random (MNAR) situation[3]. For example, consider a video website that offers two genres of movies: comedies and tragedies. Suppose we only observe the ratings for comedies and do not know whether the users have watched or rated any tragedies. Assuming MAR, we would infer that the users are not interested in tragedies and would only recommend comedies to them. However, this may not be true, as some users may prefer tragedies over comedies, but they have not rated them for specific reasons. In this case, we may miss some potential recommendations that satisfy users' preferences. This illustrates that the MAR assumption may result in a biased estimation of the user-item preferences and a self-reinforcing feedback loop that favors the items with more observed ratings over the ones with less or no ratings, known as the *Matthew Effect*. To solve this problem, some researchers [3–5] propose the MNAR assumption: the missing ratings depend on their values and other factors, such as item popularity or contextual information. MNAR assumption is more common and applicable in real-world recommendation scenarios than the MAR assumption (as shown in Figure 1).

In recent years, several methods have been developed to tackle the MNAR problem in recommender systems using various techniques, such as matrix factorization [5–9], variational autoencoders [4], and generative adversarial networks [10–13]. However, despite their advances and innovations, these methods still have limitations and challenges that hinder their applicability and effectiveness in real-world scenarios. Specifically, we identify two main issues that need to be addressed: 1) **Data Limitation**: Most MNAR methods require MAR data as ground truth for supervised learning or propensity estimation (e.g., IPS [7], DRJL [8], TPMF [9], and GINA [4]). However, collecting


- *Y. Xu, En Wang†(corresponding author), are with the Department of Computer Science and Technology, Jilin University, Changchun, 130012, China. E-mail: yuanbox@jlu.edu.cn, wangen@jlu.edu.cn.*
- *F. Zhuang is with 1. the Institute of Artificial Intelligence, Beihang University, and 2. SKLSDE, School of Computer Science, Beihang University, E-mail: zhuangfuzhen@buaa.edu.cn.*
- *C. Li is with the Key Laboratory of Trustworthy Distributed Computing and Service (MoE), Beijing University of Posts and Telecommunications. E-mail: lichaozhuo@bupt.edu.cn.*
- *J. Wu is with the Department of Computer and Information Science, Temple University, Philadephia, PA 19122, USA. E-mail: jiewu@temple.edu.*

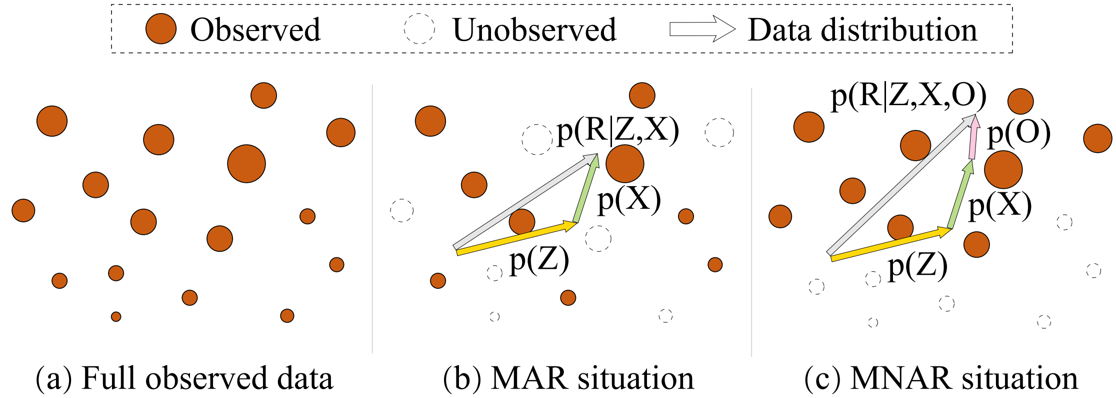



Fig. 1. A visualized example for MAR and MNAR assumption. The size of the circle denotes the rating value. The feedbacks predicted directly from observed data are biased from ground truth in MNAR situation.

MAR data is costly and impractical in most cases, as it involves asking users to rate items they have not interacted with or are not interested in. Moreover, there are only a few public datasets that provide both MAR and MNAR data for evaluation purposes (e.g., Yahoo [3] and Coat [7]), and they are relatively small-scale and domain-specific. Therefore, these MNAR methods are often validated on synthetic data or limited real-world data, which may not reflect the true complexity and diversity of the MNAR problem. Considering that most real-world datasets are MNAR, the lack of MAR data poses a significant challenge for developing and testing MNAR methods. 2) **Task limitation**: Existing MNAR methods formulate the problem as a binary matrix completion task, where the goal is to predict the implicit feedback (0/1) of users towards items based on their observed interactions. However, this formulation is too simplistic and restrictive for real-world applications, where users may have different levels of preference or satisfaction for different items, usually represented by explicit ratings (e.g., 1-5 stars). These ratings contain more information and nuances than binary feedbacks. Thus, they can help to rank the items more accurately and fairly. However, most MNAR methods ignore or discard these ratings and only focus on the binary feedbacks, which may lead to suboptimal recommendations. The MNAR problem should be considered on explicit ratings for a more comprehensive and realistic recommendation task.

Motivated by these challenges, we propose a novel approach to predict MNAR ratings without relying on MAR data. Our approach is based on the observation that user preferences are consistent and independent of the missing mechanism in both MAR and MNAR data. That is, user preferences reflect the intrinsic interest of users towards items, regardless of whether they rate them or not. By analyzing the data generation process of both MAR and MNAR data, we find that user preferences influence the observation probability and the rating value of each user-item pair. Therefore, we hypothesize that user preferences can serve as a common prior propensity for both MAR and MNAR data, and we can use them to bridge the gap between them. Based on this hypothesis, we develop a lightweight iterative probabilistic matrix factorization framework (lightIPMF) for the explicit rating prediction task. Our framework uses user preferences extracted from MNAR data instead of MAR data to estimate MNAR ratings. The model is trained, validated, and tested on four real-world MNAR datasets to demonstrate its effectiveness and robustness for practical MNAR scenarios. Moreover, our framework is data-agnostic, meaning it can be applied to either MAR or MNAR data for comparison with the state-of-the-art methods based on either assumption.

To summarize, this work makes the following important contributions:

- Presenting a data view of the consistency hidden in MAR and MNAR for recommendation and utilizing *user preference* for solving the MNAR problem without MAR prior propensity. To the best of our knowledge, it is the first work that explicitly indicates the bridge between MAR and MNAR is user preferences.
- Proposing a data-agnostic framework and a practical model: lightIPMF that trains the recommender model considering user preference and missing mechanism according to MNAR or MAR data for explicit feedbacks. lightIPMF is a realization of our proposition with three modules, which consider user preference as a common factor between MAR and MNAR data generative process, thus deducing user preference from either MNAR or MAR for explicit rating predictions.
- Evaluating the proposed model on four real-world datasets (with different areas, data distributions, and scales) to demonstrate effectiveness and rationality. We validate our proposed lightIPMF on both MAR and MNAR situations, comparing with the specific models utilizing MAR as the prior propensity. The experimental results indicate that lightIPMF outperforms the state-of-the-art MNAR recommenders.

The organization of this paper is as follows: In section 2, we give the basic definitions and propositions to build the foundations. We give the methodology and extend it to a practical solution for MNAR without MAR prior in section 3. Validations and discussions are conducted on several MAR and MNAR datasets in section 4. Related works are reported in section 5. We conclude our work in section 6.

## 2 PRELIMINARY

To build the theoretical foundation of our proposed method, we give basic definitions, including the MNAR-MAR problem and some propositions in this section.

### 2.1 Basic Definition

In a recommender system scenario with $M$ users $U$=$(u_1, u_2, ..., u_m)$ and $N$ items $I$=$(i_1, i_2, ..., i_n)$, let $R^o$ be the observed user-item rating matrix $R^o \in \mathbb{R}^{M\times N}$, whose entity $r^o_{ui}$ denotes the ratings (1,2,...,5 for explicit feedback, and $0/1$ for implicit feedback) that user $u$ rate item $i$. Because the items scale $N >>$ the user scale $M$ and each user can only rate a small part of items, $R^o$ is an extremely sparse matrix that most ratings are unobserved. Assuming there is a ground truth, the full observed matrix $R \in \mathbb{R}^{M\times N}$. And there is another prediction user-item matrix $R^c$, where we can predict all the unobserved ratings as $r^c_{ui}$ from recommender models. A typical recommend task $T$ is: for object user $u$, the recommender should minimize the Loss $\mathcal{L}_{rec}$==$\frac{1}{M\times N}E(r^c_{ui}, r_{ui})$, and then recommend the Top-$k$ high-feedback unobserved items $(i_1, i_2, ...i_k \in R^c)$ that $u$ may consume. $E(r^c_{ui}, r_{ui})$ could be $|r^c_{ui} - r_{ui}|$ for mean absolute error (MAE), or $(r^c_{ui} - r_{ui})^2$ mean square error (MSE), or other operations for other metrics.

Then we consider the MAR and MNAR situation: Let $O$ be the observation indicator matrix $O \in \mathbb{R}^{M\times N}$. For each observed rating $r^o_{ui} \in R$, $o_{ui}$=1, and for the unobserved rating $o_{ui}$=0. Let $R^o$ and $R^m$ denote the observed and missing rating matrices, respectively. And $R^o \cup R^m = R$. Similar to the notation introduced by [4], we define a probabilistic distribution $p(R)$ on $R$ as the rating distribution we would have observed if no missing mechanism was present. We define the conditional distribution $p(O|R)$ as the missing mechanism, which decides the probability of each $r_{ui}$ being missing. We also define the marginal distribution for partially observed ratings, $\log p(R^o, O) = \log \int_{R^m} p(R^o, R^m, O)\mathrm{d}R^m$. The three assumptions from the framework of [14] pertain to the specific form of this conditional distribution: If the recommendation scenario is MCAR, $p(O|R)$=$p(O)$ without any missing mechanism; if it is MAR, $p(O|R)$=$p(O|R^o)$=$p(O|R^m)$; otherwise it is MNAR, as shown in Figure 2. We treat recommend task $T$ as a matrix completion problem: given the observed rating matrix $R^o$ and the observation indicator matrix $O$, recover the unobserved ratings in $R^m$ to form an approximate rating matrix $\hat{R}^g$ for achieving $R^g$.

### 2.2 MNAR-MAR Problem Definition

From the ground truth $R$, we suppose a ground truth data generative process $p_g(R^o, O)$ where $r^o_{ui}, o_{ui}$ are partially observed. We need to optimize the parameters $(\alpha, \theta)$ of a joint generative process $p_{\alpha,\theta}(R^o, R^m, O)$, where $p_\alpha(R)$ is the rating distribution and $p_\theta(O|R)$ is the missing mechanism. When missing data is MCAR or MAR, the missing mechanism can be ignored when performing maximum likelihood (ML) inference based only on the observed data, as Formula (1):

$$\begin{aligned}&\arg\max_{\alpha} \mathbb{E}_{(r^o_{ui},o_{ui})\sim p_g(R^o,O)} \log p_\alpha(R^o = r^o_{ui}) = \\ &\arg\max_{\alpha} \mathbb{E}_{(r^o_{ui},o_{ui})\sim p_g(R^o,O)} \log p_\alpha(R^o = r^o_{ui}, O = o_{ui}).\end{aligned} \tag{1}$$

Note that $\log p(R^o) = \log \int_{R^m} p(R^o, R^m)dR^m$, and we make realization of $R^o$ and $O$: $(r^o_{ui}, o_{ui}) \sim p_g(R^o, O)$. The EM algorithm or other optimization methods can solve this in practice. However, when considering the MNAR situation, this argument does not hold. Check Figure (c) in 2 that $R$ is the cause of $O$, which happens in most scenarios of recommender systems. All the $o_{ui}$ in $O$ are conditionally independent of each given $R$. To modify this generating process by considering the missing mechanism, existing work [14] jointly learns both rating distribution $p_\alpha(R)$ and missing mechanism $p_\theta(O|R)$ by maximizing:

$$\begin{aligned}\arg\max_{\alpha,\theta} \mathbb{E}_{(r^o_{ui},o_{ui})\sim\, p_g(R^o,O)}[&\log p_\alpha(R^o = r^o_{ui}) \\ &+ \log p_\theta(O = o_{ui}\,|R^o = r^o_{ui}\,)].\end{aligned} \tag{2}$$

This factorization is called the selection modeling [6, 10]. There are multiple challenges for utilizing Formula (2) to make unbiased data completion for recommendations. First, for various MNAR scenarios, achieving the model assumptions consistent with $p_g(R^o, O)$ in real-world situations is difficult. Second, some algorithm utilizes learning prior propensity from a small MAR dataset to optimize the objective function, ignoring that collecting MAR in real-world scenarios is complicated. Third, most researchers only focus on MNAR matrix completion and validate their models on synthetic MNAR datasets, limiting these algorithms' application range.

### 2.3 Propositions For MNAR-MAR Problem

Different from other existing MNAR models that focus on the model framework or optimizations directly, we first give some propositions from data perspectives for solving the challenges above:

**Proposition 1** *(Data consistency): There exists a consistency across MAR and MNAR datasets, which can be treated as a common prior propensity $X$ for MAR and MNAR data generative procedure, and benefits existing recommenders' performance.*

We first investigate MNAR and MAR datasets with the state-of-the-art MNAR algorithm (MF-IPS [7]) to validate the proposition. Here is a reminder for MF-IPS: this algorithm learns a prior propensity $p_{ui} > 0$ for all $u, i$, and estimates matrix factorization with IPS-estimator $p_{ui}$ to achieve "unbiased" data completion. We build two variants of MF-IPS: MF-IPS-MAR and MF-IPS-MNAR, which learn their prior propensity $p_{ui}$ from MAR and MNAR datasets, respectively. As shown in Figure 3, we notice that MF-IPS-MAR and MF-IPS-MNAR can both enhance MF on recommendation metrics, indicating that both prior propensities learned simply from MAR and MNAR benefit the recommenders' performance. From this phenomenon, we boldly hypothesize that there exists a common prior propensity $X$ among MAR and MNAR datasets, and utilizing this common prior propensity from MNAR may improve recommendation performance. Then we give the second proposition:

**Proposition 2** *(Data Construction): MNAR and MAR distributions are both based on MCAR distribution, part-independently. MNAR can be treated as MAR with common prior propensity $X$ for generating observation $O$, and MAR can be treated as MCAR with common prior propensity $X$ and observed rating $R^o$.*

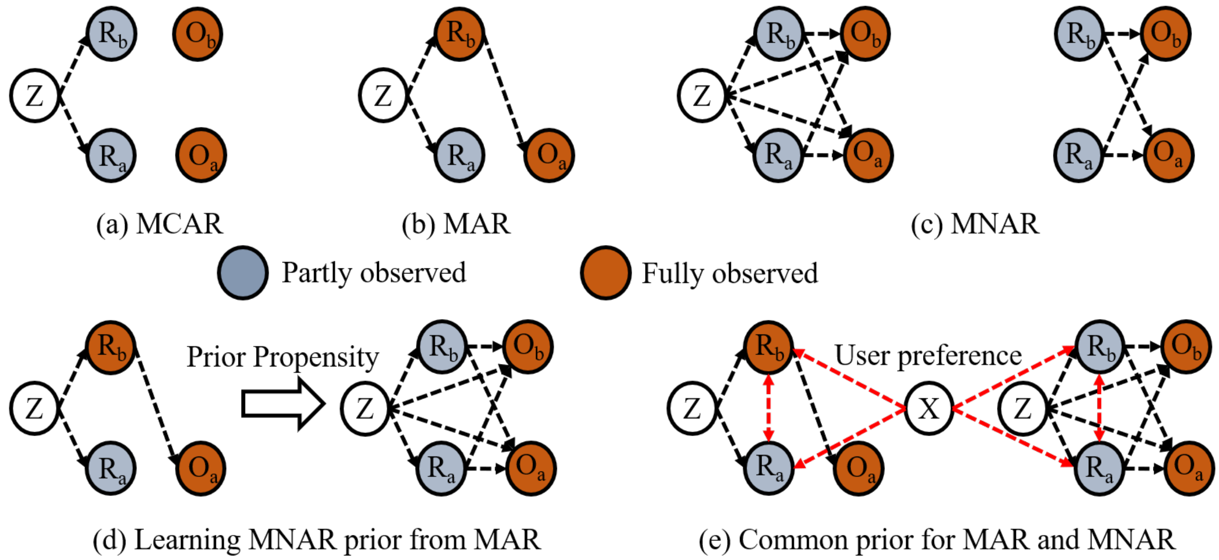



Fig. 2. MCAR-(a), MAR-(b), and MNAR-(c). (d) illustrates heuristic supervised learning methods (MF-IPS, MF-DRJL, etc) for MNAR. (e) illustrates our theory that user preference $X$ generates ratings $R$ in both MNAR and MAR, jointly with latent prior $Z$. $R_a$, $R_b$ denote different sub-datasets sampled from $R$.

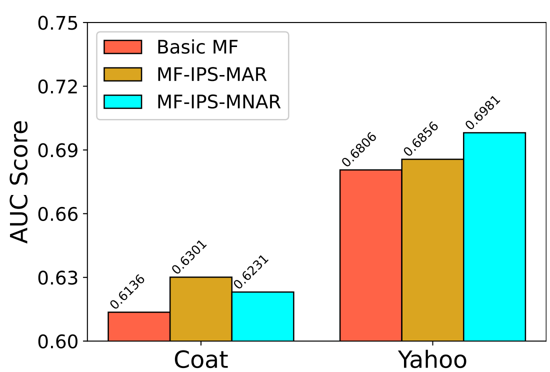



Fig. 3. AUC Score for Coat and Yahoo. The prior propensity $P$ learned from either MAR or MNAR can provide performance gain on both datasets, which proves Proposition 1.

Following our proposition, the joint distribution of $p(R, O)$ is formulated as:

$$\begin{aligned} p(R,O) &= p(R|Z)p(O); && \text{MCAR} \\ p(R,O) &= p(R|Z,X)p(O|R^o); && \text{MAR} \\ p(R,O) &= p(R|Z,X)p(O|R^o,X,Z); && \text{MNAR} \end{aligned} \tag{3}$$

Note that latent prior $Z$ can be treated as natural characters or domain-specific factors affecting ratings, which could be learned from side information. To obtain the common prior propensity $X$, we analyze the construction of MNAR and MAR datasets, including Yahoo [3] and Coat [7]. Inspired by [2, 3], we notice that MAR is an unbiased dataset with randomly selected items and users' feedbacks, while the MNAR is with user-select items and their feedbacks. Note that *user preference* is the only stable character that occurs in both MAR and MNAR data construction periods. Thus, we give the last proposition:

**Proposition 3** *(User Preference in MNAR). The common prior propensity $X$ can be treated as user preferences, which are stable in both MAR and MNAR.*

Moreover, user preference $X$ decides the feedbacks of items ($R^o$), and it is the most critical factor for a recommender system to understand users' interactions, patterns, and habits. With the propositions above, deducing user preference can unify the objective functions of the MNAR task and recommendation task. MAR data is complicated to obtain (only Yahoo and Coat for the public), while MNAR data is standard in real-world scenarios. Our goal is *to obtain user preference $X$ from only the MNAR dataset to solve the MNAR problem and achieve an accurate, explicit-rating recommendation.*

# 3 Methodology for Solving MNAR without MAR

## 3.1 MNAR Joint Distribution with User Preference

Inspired by the research in [4, 9], we propose a novel joint distribution for explicit MNAR rating data by extracting user preference as the common prior propensity of MAR and MNAR for recommendations. We devise three generative modules in this joint distribution: 1) a rating prediction model (RPM) to estimate $R$ with parameter $\alpha$. 2) an observation prediction model (OPM) to estimate $O$ with parameter $\theta$, and 3) a user preference model (UPM) to estimate $X$ with parameter $\beta$. Specifically, the joint distribution of $R, O, X$ for MAR/MNAR with parameters $\alpha, \theta, \beta$ is formulated as:

$$\begin{aligned} p_{\text{MAR}}(R,O,X|\Theta) &= p_\alpha(R|X,Z)p_\theta(O|R^o)p_\beta(X|R^o), \\ p_{\text{MNAR}}(R,O,X|\Theta) &= p_\alpha(R|X,Z)p_\theta(O|R^o,X,Z)p_\beta(X|R^o), \end{aligned} \tag{4}$$

where $\Theta$ denotes the parameter space $\alpha \times \theta \times \beta$. The joint distribution formulation's motivation is that the full-observed ratings $R$ are first generated by RPM with latent prior $Z$ and user preference $X$, which are the common prior propensity of MAR and MNAR. Considering the MNAR situation, the missing mechanism $p(O|R)$ is extended by OPM, where the observation is generated by observed ratings $R^o$, $X$, and $Z$. Among this procedure, user preference $X$ is stable and can be deduced from $R^o$ by UPM. Note that this joint distribution follows the MNAR assumption that the observation is related to ratings and considers the effect of user preference according to the Propositions above.

The joint distribution of the three modules represents the relations among ratings, observations, and user preferences. This generative framework is lightweight and flexible because we can modify each module independently with different restrictions or specifications, MNAR or MAR. Or we can pre-train the three modules respectively and combine them in different ways to achieve a self-adapting recommendation. In the MNAR assumption, the user preference $X$ is shared by RPM and OPM, connecting the ratings and observations to solve the MNAR issue without MAR data, which is the most important improvement over other MNAR methods.

### 3.2 The lightIPMF: A practical model for MNAR problem in recommender systems

Then we instantiate a practical, specified lightweight iterative probabilistic matrix factorization (lightIPMF) for tackling the MNAR recommendation tasks by modeling the three modules of the joint distribution: User Preference Model (UPM), Observation Prediction Model (OPM), and Rating Prediction Model (RPM), respectively.

#### 3.2.1 User Preference Model

We factorize user preference $X$ by the observed rating matrix $R^o$. In UPM, we first utilize a user-specific threshold $t_u$ for extracting user preference $x_{ui}$ to build $X$:

$$\begin{aligned} &x_{ui} = 1, \text{ if } r_{ui} \geq t_u; \\ &x_{ui} = 0, \quad \text{else}, \end{aligned} \tag{5}$$

where $t_u = \text{avg}(r_{ui}|r_{ui} \in R^o)$ or $\text{median}(r_{ui}|r_{ui} \in R^o)$. Then we employ a probabilistic matrix factorization on $R^o$ and $X$, with latent low-rank matrices $U^o, G^x \in \mathbb{R}^{m\times k}, V^o, H^x \in \mathbb{R}^{n\times k}, k < \min(m,n)$, respectively:

$$R^o = U^o(V^o)^{\mathrm{T}}, X = G^x(H^x)^{\mathrm{T}} \tag{6}$$

Note that we use explicit feedback ($r_{ui}$ from 1 to 5), not implicit feedback (0/1) in $R^o$. Hence, $U^o$ and $V^o$ follow a zero-mean spherical Gaussian distribution. And $X$ is a binary user preference matrix. $G^x$ and $H^x$ follow a truncated standard normal distribution. We further model $x_{ui}$ with a Gaussian distribution:

$$p_\beta(X|R^o) = \prod_u^m \prod_i^n N(x_{ui}|\hat{x}_{ui}, \sigma_x^2). \tag{7}$$

$$\hat{x}_{ui} = G_u^x(H_i^x)^{\mathrm{T}} + w_u^o\left[\!\left[U_u^o(V_i^o)^{\mathrm{T}} - t_u\right]\!\right] + b^x, \tag{8}$$

where $[\![a]\!]$ returns 1 if $a \geq 0$, otherwise it returns 0; $w_u^o \in (0,1)$ is a user-specific adjustable parameter for observed ratings, $b^x$ is a bias set, and $\beta$ denotes $G^x$, $H^x$, $U^o$, $V^o$, $\sigma_x$, $w_u^o$, and $b^x$. Obviously, an item with a high rating estimation $r_{ui}^o$ is more likely to attract the users (a high rating estimation $r_{ui}^o$ denotes a high user preference $x_{ui}$ ).

#### 3.2.2 Observation Prediction Model

We define user observation $o_{ui}$ for user $u$ on item $i$:

$$\begin{aligned} &o_{ui} = 1, \text{ if } r_{ui} \neq null; \\ &o_{ui} = 0, \quad \text{else}, \end{aligned} \tag{9}$$

Inspired by [15, 16], and [9], we model binary observation indicator matrix $O$ as a Bernoulli distribution whose mean is drawn from a Beta distribution. However, existing methods do not consider the MAR or MNAR assumption. Specifically, $O$ for MAR is formulated as:

$$p_\theta(O|R^o) = \prod_u^m \prod_i^m B(o_{ui}|f(r_{ui}^o)), \tag{10}$$

where $f(r_{ui}^o)$ is a linear function to map $r_{ui}^o$ into a scalar between (0,1). For the MAR assumption, we learn $f(r_{ui}^o)$ from MAR data or a Beta distribution. However, when considering the MNAR situation, the distribution deduced from $R^o$ is usually biased from the real distribution. In our proposed framework, we jointly consider user preference, observed ratings, and latent prior propensity (i.e., $X, R^o, Z$). For simplicity and to avoid overfitting, we first estimate $\hat{r}_{ui}^o$ with user preference $X$ and latent prior $Z$, then we formulate $o_{ui}$ as a Gaussian distribution:

$$p_\theta(O|R^o, X, Z) = \prod_u^m \prod_i^n N(o_{ui}|f(\hat{r}_{ui}^o), \sigma_o^2), \tag{11}$$

$$\hat{o}_{ui} = U_u^o(V_i^o)^{\mathrm{T}} + w_u^x G_u^x(H_i^x)^{\mathrm{T}} + z_i + b^o, \tag{12}$$

where $w_u^x \in (0,1)$ is a user-specific adjustable parameter for user preference, $b^o$ is a biased set, $z_i$ is a latent prior (it can be treated as a scalar learned from the nature characters of item $i$), and $\theta$ denotes $G^x$, $H^x$, $U^o$, $V^o$, $\sigma_o$, $w_u^x$, and $b^o$. Intuitively, this distribution indicates that the user preference and nature characters do affect the observation, which holds the MNAR assumption.

#### 3.2.3 Rating Prediction Model

First, we factorize $R$ by two low-rank latent matrix $U^r \in \mathbb{R}^{m\times k}$ and $V^r \in \mathbb{R}^{n\times k}$, representing latent user attributes and item attractions respectively. We model $R$ with the Gaussian distribution below:

$$p_\alpha(R|X,Z) = \prod_u^m \prod_i^n N(r_{ui}|\hat{r}_{ui}, \sigma_r^2) \tag{13}$$

$$\hat{r}_{ui} = U_u^r(V_i^r)^T + c_x G_u^x(H_i^x)^T + z_i + b^r, \tag{14}$$

Moreover, learning our proposed models from MNAR data is an unbiased model from MAR data because the full-observed rating matrix $R$ is only based on $X$ and $Z$, not the observation $O$.

### 3.3 Optimization for lightIPMF

#### 3.3.1 Loss Function for Joint Distribution Model of MNAR

With the three models (UPM, OPM, and RPM), we can achieve ratings, user preferences, and observations by specifying Formula (2) to log joint probability:

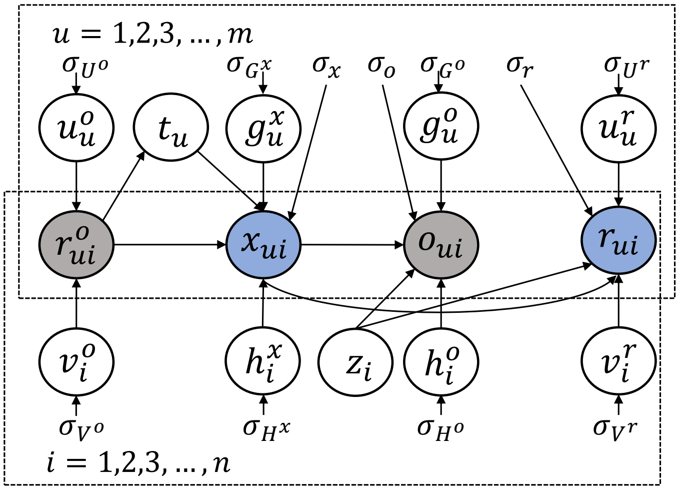



Fig. 4. Visualized representation of lightIPMF. It contains three models: UPM, OPM, and RPM. $r^o_{ui}$ and $o_{ui}$ (dark circle) are partly observed, and $x_{ui}$ and $r_{ui}$ (blue circle) are totally unobserved, which could be predicted.

$$
\begin{aligned}
L_{\mathrm{MNAR}} = \\
\log p(R, O, X|\Theta) = \sum_u^m \sum_i^n \log N(r_{ui}|\hat{r}_{ui}, \sigma_r^2) \quad (15)\\
+ \log N(o_{ui}|f(\hat{o}_{ui}), \sigma_o^2) + \log N(x_{ui}|\hat{x}_{ui}, \sigma_x^2).
\end{aligned}
$$

Note that the three modules share some variables and parameters in MNAR situations.

#### 3.3.2 Iterative Joint Learning with Explicit Feedback Recommendation

We could utilize classic Expectation-Maximization (EM) to find the maximum posterior estimates of the parameters $\Theta$. However, because some parameters are shared among three models, we devise an iterative learning procedure, which can be treated as a variant of the EM algorithm:

**E-step**: Observed rating $R^o$ and Observation indicator $O$ are not full observed. After initializing parameters, we 1) calculate the missing rating $\hat{r}^o_{ui} \in R^m$, 2) use $\hat{r}^o_{ui}$ to calculate $\hat{x}_{ui}$, 3) use $\hat{r}^o_{ui}$, $\hat{x}_{ui}$ to calculate $\hat{o}_{ui}$. Note that we update the parameters and obtain full-observed ratings $\hat{r}_{ui}$ in M-step by using $R^o$ as supervision. Specifically, we obtain the expectation of unknown $o_{ui}$:

$$
\mathbb{E}(o_{ui}|\hat{r}^o_{ui}, \hat{x}_{ui}, z_i) = \frac{z_i \cdot N(0|\hat{r}^o_{ui}, \sigma_x^2) \cdot N(0|\hat{x}_{ui}, \sigma_x^2)}{z_i \cdot N(0|\hat{r}^o_{ui}, \sigma_x^2) \cdot N(0|\hat{x}_{ui}, \sigma_x^2) + (1 - z_i)}. \quad (16)
$$

Considering the explicit feedbacks, we divide the continuous estimation $\hat{r}^o_u i$ into 5 levels with function $EL(\hat{r}^o_{ui})$, similar to [9]:

$$
\mathbb{E}(o_{ui}|\hat{r}^o_{ui}, \hat{x}_{ui}, z_i) = \frac{z_i \cdot EL(\hat{r}^o_{ui}) \cdot N(0|\hat{x}_{ui}, \sigma_x^2)}{z_i \cdot EL(\hat{r}^o_{ui})) \cdot N(0|\hat{x}_{ui}, \sigma_x^2) + (1 - z_i)}. \quad (17)
$$

**M1-Step**: while in M1-step, we first calculate full-observed rating $\hat{r}_{ui}$ with $\hat{r}^o, \hat{x}, \hat{o}$. Because all the parameters are involved in Formula (17), we set $q_{ui}$=$\mathbb{E}(o_{ui}|\hat{r}^o_{ui}, \hat{x}_{ui}, z_i)$, and update the parameters of $x_{ui}$ as below:

$$
\begin{aligned}
G^x_u &\leftarrow (\sigma_x \sum_i^n q_{ui} G^x_u (G^x_u)^{\mathrm{T}} + \sigma_{G^x_u} I_k)^{-1} (\sum_i^n \sigma_x q_{ui} x_{ui} H^x_i);\\
H^x_i &\leftarrow (\sigma_x \sum_u^m q_{ui} H^x_i (H^x_i)^{\mathrm{T}} + \sigma_{H^x_i} I_k)^{-1} (\sum_u^m \sigma_x q_{ui} x_{ui} G^x_u),
\end{aligned} \quad (18)
$$

where $\sigma_{G^x_u}$, $\sigma_{H^x_i}$ are the parameters of $G^x_u$ and $H^x_i$.

**M2-Step**: We fix the parameters in M1-Step, calculate $q_{ui}$, and update the parameters of $r^o_{ui}$:

$$
\begin{aligned}
U^o_u &\leftarrow (\sigma_o \sum_i^n q_{ui} U^o_u (U^o_u)^{\mathrm{T}} + \sigma_{U^o_u} I_k)^{-1} (\sum_i^n \sigma_o q_{ui} r^o_{ui} V^o_i);\\
V^o_i &\leftarrow (\sigma_o \sum_u^m q_{ui} V^o_i (V^o_i)^{\mathrm{T}} + \sigma_{V^o_i} I_k)^{-1} (\sum_u^m \sigma_o q_{ui} r^o_{ui} U^o_u),
\end{aligned} \quad (19)
$$

where $\sigma_{U^o_u}$, $\sigma_{V^o_i}$ are the parameters of $U^o_u$ and $V^o_i$. Note that we should utilize M1-step and M2-step iteratively, so the optimization should be $E \rightarrow M1 \rightarrow M2 \rightarrow M1 ... \rightarrow convergence$. Then we achieve the parameter set $\Theta$, and achieve a full user-item explicit rating matrix $R$.

## 4 Validations and Discussions

In this section, we conduct extensive experiments to answer the following research questions:

- **RQ1:** Does lightIPMF outperform existing MNAR algorithms, including some methods with MAR data as priors?
- **RQ2:** How does user preference $X$ solve the MNAR problem without any MAR data as the ground-truth prior propensity for training?
- **RQ3:** How do different components in lightIPMF contribute to the recommendation performance?
- **RQ4:** How do different parameter settings affect the recommendation performance? Is lightIPMF an efficient model?

### 4.1 Experiment Settings

#### 4.1.1 Datasets

The comprehensive evaluation should be verified on different data assumptions (MAR or MNAR). Two real-world datasets with MAR ratings are considered: 1) Yahoo R3 (denoted Yahoo) [3] collects 311, 704 MNAR ratings and 45, 000 MAR ratings from 15,400 users on 1, 000 songs. 2) The Coat (Coat) [8] has 6, 960 MNAR ratings and 4, 640 MAR ratings of 290 users to 300 coats. Additionally, we use two widely-used datasets ML10M[1] and Amazon Beauty (Amazon for short) [2]. Note that 1) Yahoo and Coat are the only public datasets with MAR and MNAR. 2) Movielens and Amazon only contain MNAR ratings. All the datasets above are publicly available and vary in terms of domain, size, and sparsity. The statistics of these datasets are summarized in Table 2.

1. http://grouplens.org/datasets/movielens/
2. http://jmcauley.ucsd.edu/data/amazon/links.html

TABLE 1
Comparison between different MNAR models.

| Model | MNAR | Prior Propensity | User Preference | Estimated Bias | Train/Test set | Basic Model |
|---|---|---|---|---|---|---|
| PMF | × | × | × | × | MAR/MAR | Matrix Factorization |
| MF-MNAR | ✓ | × | × | × | MAR/MNAR | Matrix Factorization |
| MF-IPS | ✓ | ✓ | × | × | MAR/MNAR | Matrix Factorization |
| MF-DRJL | ✓ | ✓ | × | ✓ | MAR/MNAR | Matrix Factorization |
| TPMF | ✓ | × | ✓ | × | MAR/MNAR | Matrix Factorization |
| PVAE | × | × | × | ✓ | MNAR/MAR | Variational Autoencoders |
| not-MIWAE | ✓ | × | ✓ | × | MNAR/MAR | Variational Autoencoders |
| GINA | ✓ | × | × | ✓ | MNAR/MAR | Variational Autoencoders |
| lightIPMF | ✓ | ✓ | ✓ | × | MNAR/MNAR | Matrix Factorization |

TABLE 2
Datasets Statistics of four different datasets.

| Datasets | *#Users* | *#Items* | *#MNAR* | *#MAR* |
|---|---|---|---|---|
| Yahoo | 15, 400 | 1, 000 | 311, 704 | 45, 000 |
| Coat | 290 | 300 | 6, 960 | 4, 640 |
| ML10M | 69, 166 | 8, 790 | 5, 000, 415 | - |
| Amazon | 6, 403, 006 | 1, 660, 119 | 14, 771, 988 | - |

#### *4.1.2 Evaluation*

We report all ranking performance w.r.t. three widely used metrics: MSE, AUC, and Normalized Discounted Cumulative Gain NDCG cut at $K$ (we set $K$=10 without additional explanation). Note that the conventional evaluation strategy of the MNAR model focuses on implicit feedback, not explicit feedbacks. Consequently, the test model can perform well over 0/1 with the MSE metric. Moreover, we use explicit feedback directly (rating 1 to 5), not implicit feedback (0/1), for validation. Following the [4, 9], we randomly leave 10% feedbacks as validation data, 10% as test data, and all the others as training data. Specifically, in the MNAR situation, we set $o_{ui} = 1$ for observed items with feedbacks, otherwise $o_{ui} = 0$; $x_{ui} = 1$ for the items with $r_{ui} \geq t_u$, otherwise $x_{ui}$=0. We set $t_u$ as the average ratings of user $u$. For the datasets (ML10M and Amazon) without MAR data, we treat its training set or test set as the MAR data to meet the data requirement listed in Table 1. We tune the hyperparameters on validation sets by grid search for a fair comparison and obtain the best for testing. We implement GS$^2$-RS based on Pytorch accelerated by NVIDIA RTX 3090 GPU. The core code is available at *https://github.com/uuthx/bias_exeperiment_code.git*.

#### *4.1.3 Baselines*

We compare our proposed method with the following baselines:

- **PMF [17]**: PMF is the representative, classical model for recommendations.
- **MF-MNAR [5]**: MF-MNAR is the first viable matrix factorization method considering the MNAR data assumption.
- **MF-IPS [7]**: MF-IPS adds the standard Inverse Propensity Weight to reweight samples for unbiased recommendations.
- **MF-DRJL [8]**: MF-DRJL proposes a more robust unbiased estimator by integrating inverse propensity score and estimated imputed errors for the MNAR rating data.
- **TPMF [9]**: TPMF considers MNAR ratings by exploring the complex dependencies between item observability, user selection, and ratings.
- **PVAE [4]**: PVAE is a probabilistic model to build a generative model for recommendation with MAR assumption.
- **Not-MIWAE [10]**: Not-MIWAE is a deep latent variable model (DLVM) proposed for inference and imputation in missing data problems where the missing mechanism is MNAR.
- **GINA [4]**: GINA is a state-of-the-art practical algorithm model based on VAEs, which applies flexible deep generative models in a principled way for MNAR problems.

Most MF-based MNAR models consider prior propensity or user preference separately and utilize MAR for calculating the prior propensity of MNAR. VAE-based models directly generate the MNAR feedbacks to deduce MAR feedbacks. The comparison between different MNAR models is summarized in Table 1.

### 4.2 Experimental Results

#### *4.2.1 Overall Performance (RQ1)*

To analyze the effectiveness of lightIPMF, we evaluate lightIPMF on the four real-world datasets compared with the baselines. The results are reported in Table 3. The observation and analysis are:

- Our proposed lightIPMF outperforms all the baselines under all the metrics on all the datasets. Specifically, we notice that some methods which require MAR as the prior propensity (PMF, MF-MNAR, MF-IPS, MF-DRJL, and TPMF) perform well on Yahoo and Coat. Nevertheless, without MAR prior propensity (ML10M and Amazon), the prediction accuracy of ratings are declining (MSE and AUC) more than VAE-based methods (PVAE, not-MIWAE, and GINA). The results are well explainable: MF-based models cannot estimate accurate ratings without the propensity in MAR (or a biased prediction for the propensity from MNAR). In comparison, the VAE-based methods can implicitly obtain the knowledge hidden in MNAR to enhance recommendations.

TABLE 3
MSE, AUC and NDCG on real-world datasets, where the best ones are in bold. We evaluate the method with top-10 recommended items, and IMP(%) denotes the performance gain upon best baselines (underline).

| Dataset | Metric | PMF | MF-MNAR | MF-IPS | MF-DRJL | TPMF | PVAE | not-MIWAE | GINA | lightIPMF | IMP.(%) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Yahoo | MSE↓ | 1.495 | 1.433 | 1.476 | 1.391 | 1.381 | 1.411 | 1.372 | <u>1.360</u> | **1.295** | *4.78* |
| | AUC↑ | 0.677 | 0.673 | 0.679 | 0.689 | 0.690 | 0.694 | 0.699 | <u>0.701</u> | **0.721** | *2.86* |
| | NDCG↑ | 0.644 | 0.649 | 0.760 | 0.770 | 0.773 | 0.764 | 0.774 | <u>0.781</u> | **0.830** | *6.41* |
| Coat | MSE↓ | 0.872 | 0.808 | 0.859 | 0.804 | 0.799 | 0.812 | 0.739 | <u>0.744</u> | **0.714** | *4.03* |
| | AUC↑ | 0.702 | 0.708 | 0.715 | 0.691 | 0.711 | 0.709 | 0.732 | <u>0.801</u> | **0.844** | *4.12* |
| | NDCG↑ | 0.684 | 0.688 | 0.689 | 0.679 | 0.691 | 0.688 | 0.711 | <u>0.734</u> | **0.754** | *2.72* |
| ML10M | MSE↓ | 1.898 | 1.746 | 1.766 | 1.713 | 1.721 | 1.689 | 1.701 | <u>1.674</u> | **1.623** | *3.04* |
| | AUC↑ | 0.567 | 0.579 | 0.581 | 0.574 | 0.583 | 0.581 | <u>0.591</u> | 0.589 | **0.621** | *5.07* |
| | NDCG↑ | 0.533 | 0.545 | 0.543 | 0.561 | 0.555 | 0.562 | 0.566 | <u>0.569</u> | **0.651** | *14.6* |
| Amazon | MSE↓ | 2.141 | 2.008 | 1.974 | 1.964 | 1.954 | 1.875 | 1.846 | <u>1.833</u> | **1.801** | *1.74* |
| | AUC↑ | 0.541 | 0.534 | 0.543 | 0.551 | 0.567 | 0.571 | <u>0.601</u> | 0.599 | **0.661** | *10.4* |
| | NDCG↑ | 0.504 | 0.513 | 0.521 | 0.524 | 0.531 | 0.534 | 0.547 | <u>0.561</u> | **0.613** | *9.27* |

However, these VAE-based methods can not explicitly model the knowledge in MNAR introduced by our proposed framework.

- Moreover, we can compare the different methods across different datasets. We notice that even using MNAR as MAR for computing prior propensity, performance gain still exists (compare PMF with MF-MNAR, MF-IPS, and MF-DRJL on ML10M and Amazon). The performance gain is not as evident as they occur on Yahoo and Coat, but it still indicates that the prior propensity in MAR and MNAR have some similarities, and utilizing the prior propensity does improve the recommendation performance. Combining with the former experiment result shown in Figure 3, it proves the fact that user preference is the common prior propensity in both MAR and MNAR. Finally, as we explicitly consider modeling user preference $X$ for ratings, lightIPMF can predict accurate ratings, enhancing NDCG with an average of 10%

#### 4.2.2 Preference Study (RQ2)

We study user preference for solving MNAR problems. We set three user preferences: Noisy preference $X_{NOISE}$, generated from a standard Gaussian distribution. Two user preferences, $X_{MAR}$ and $X_{MNAR}$, are generated from MAR and MNAR, respectively. We combine user preference with three classical methods: PMF, MF-IPS, and not-MIWAE. We conduct experiments on Yahoo and Coat with MSE, AUC, and NDCG, as shown in Figure 5 and Figure 6. The observation and analysis are:

- The results indicate that considering user preference $X$ can enhance the recommendation performance comprehensively. In detail, we notice that on larger data set Yahoo, using $X_{MAR}$ does improve the MSE, AUC, and NDCG over using $X_{MNAR}$, $X_{NOISE}$, or original models. The reason is that MAR offers unbiased user preferences for a recommendation. Specifically, for not-MIWAE, $X_{NOISE}$ hurts its performance worse than other models. It reveals a situation that for a VAE-based model, an inaccurate prior propensity may lead to a vital performance reduction.
- Obviously, models with $X_{NOISE}$ may hurt their performance. Models with $X_{MAR}$ achieve their best performance because the user preference is modeled accurately. However, although $X_{MNAR}$ cannot be as accurate as $X_{MAR}$, it still improves the performance, validating user preference's effect for MNAR recommendation tasks.

#### 4.2.3 Ablation Study (RQ3)

In this section, we first study each model's effect in lightIPMF (UPM, OPM, and RPM). We build three variants of lightIPMF: NUlightIPMF: original model without user preference model; NOlightIPMF: original model without observation prediction model; and NRlightIPMF: original model without rating prediction model. Also, we build a variant whose user preference $X$ is learned from MAR: MAlightIPMF. The ablation study results are reported in Table 4. The observation and analysis are:

- From the results of two MNAR/MAR benchmark datasets Coat and Yahoo, we notice that lightIPMF and MAlightIPMF perform better than other variants. Specifically, MAlightIPMF utilizes unbiased MAR for extracting user preferences, which obtains the performance gain on the Coat dataset. However, it performs worse than lightIPMF on Yahoo, which is an interesting phenomenon. We consider it as an over-clean situation. In the MAR dataset, we consider all the biases are removed. However, as our final goal, the recommendation should be proposed by considering multiple factors, including user preferences and even some bias, especially for large datasets [18, 19]. These factors exist in MNAR rather than MAR datasets. Our lightIPMF extracts user preference from MNAR, where these user preferences $X$ may contain some helpful bias for recommendations, thus leading to better performance.
- Note that the original lightIPMF achieves the best performance among its variants except MAlightIPMF. Especially, NUlightIPMF can be treated as a traditional IPS-based method, while NOlightIPMF can be treated as a basic PMF method. Also, MAlightIPMF has a better performance on Coat lightly

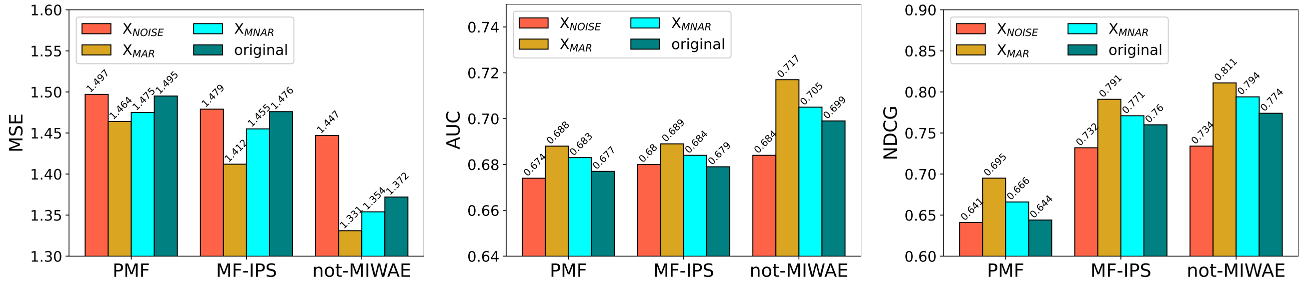



Fig. 5. Preference Study on Yahoo, validated with MSE↓, AUC↑, and NDCG↑

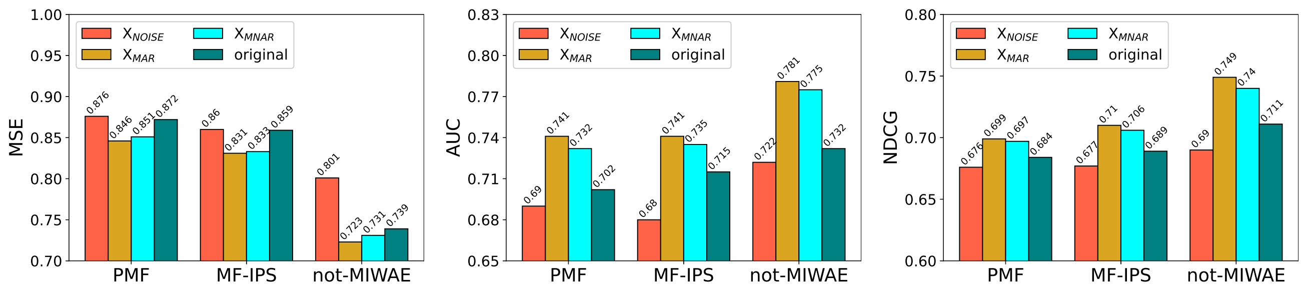



Fig. 6. Preference Study on Coat, validated with MSE↓, AUC↑, and NDCG↑

over lightIPMF. Considering the trade-off between the difficulty of obtaining MAR datasets and recommendation accuracy, we claim lightIPMF can tackle the MNAR situation properly.

#### 4.2.4 Parameter and Efficiency Analysis (RQ4)

We study latent dimension $k$ and user-specific threshold $t_u$ in lightIPMF. We validate $k$ from $(8, 16, 32, 64)$ and $t_u$ from $(random, median, avg, mode)$ on Yahoo and Coat on NDCG and MSE. Specifically, $random$ selects a rating randomly from user $u$'s ratings as $t_u$, $median, avg, mode$ denotes user $u$'s rating's median rating, average rating, and mode rating, respectively. The parameter analysis results are reported in Figure 7. The observation and analysis are:

- We notice that the latent dimension of lightIPMF affects the performance lightly after increasing over 16. Different $t_u$ affects our model's performance ($random$ worst, $median$ best). Because the dataset is sparse, $median$ properly represents the user's preference. When the dataset is dense, we think $avg$ and $median$ may perform similarly.
- Note that lightIPMF achieves best NDCG when $k = 16$ and $t_u$=$median$. A too small or large $k$ may result in underfitting or overfitting. And $median$ can well-model user's preferences on an item, which benefits lightIPMF.

## 5 Related Work

Several related approaches address the MNAR problem [2, 20, 21]. Among these, one of the most prominent approaches is a counterfactual technique that reweighs the collected data for expectation-unbiased learning using inverse propensity score (IPS) [7]. The IPS estimator can unbiasedly estimate the loss function of interest using the biased rating feedback by weighting each sample by the inverse of its propensity score. MF-MNAR [5] is an MF model for binary matrices that models the observation probability of a matrix entry as a function of the entry's value. MF-DRJL [8] combines the EIB and IPS estimators by using both imputed errors and propensities. Some researchers focus on estimation methods and PPCA under an MNAR missing mechanism has been studied by [22]. Low-rank models are employed for estimation and imputation in MNAR settings by [23] and [6]. [9] propose a tripartite CF (TCF) framework that jointly models the triple aspects of rating generation and estimates the MNAR rating. The advantages of these methods are theoretically justified and empirically verified to outperform naive methods based on the unrealistic MCAR assumption. Besides, these methods also demonstrate the diversity and complexity of the MNAR problem and the challenges of developing effective and efficient models for learning from incomplete data.

Having a model that can learn from incomplete data expands the application range of deep learning algorithms and facilitates downstream tasks such as data imputation, which is still an active and challenging research area [24–26]. CPT-v [3] is a simple missing data model that assumes that the probability of observing a rating depends solely on the underlying rating value. [27] proposes an information-theoretic counterfactual variational information bottleneck (CVIB) as an alternative method for debiasing learning without MAR data. Deep latent variable models (DLVMs) are generative models that can map complex raw input to a flexible latent representation and have recently attracted attention in handling partially-observed data due to the advantages of generative modeling and representation learning. To overcome the intractable posterior of DLVMs, variational autoencoder (VAE) employs deep neural networks to approximate the posterior and maximizes the variational evidence lower bound (ELBO). Scalable methods

TABLE 4
Ablation study of lightIPMF, where the best performance is in bold.

| Model | Yahoo | | | Coat | | | ML10M | | | Amazon | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | MSE↓ | AUC↑ | NDCG↑ | MSE↓ | AUC↑ | NDCG↑ | MSE↓ | AUC↑ | NDCG↑ | MSE↓ | AUC↑ | NDCG↑ |
| NUlightIPMF | 1.380 | 0.690 | 0.771 | 0.800 | 0.721 | 0.695 | 1.821 | 0.595 | 0.543 | 1.965 | 0.554 | 0.521 |
| NOlightIPMF | 1.488 | 0.664 | 0.641 | 0.887 | 0.712 | 0.690 | 1.990 | 0.569 | 0.544 | 1.979 | 0.544 | 0.510 |
| NRlightIPMF | 1.401 | 0.701 | 0.746 | 0.814 | 0.713 | 0.690 | 1.699 | 0.594 | 0.574 | 1.901 | 0.561 | 0.531 |
| MAlightIPMF | 1.301 | 0.719 | 0.829 | 0.722 | **0.849** | **0.758** | - | - | - | - | - | - |
| lightIPMF | **1.295** | **0.721** | **0.830** | **0.714** | 0.844 | 0.754 | **1.623** | **0.621** | **0.651** | **1.801** | **0.661** | **0.613** |

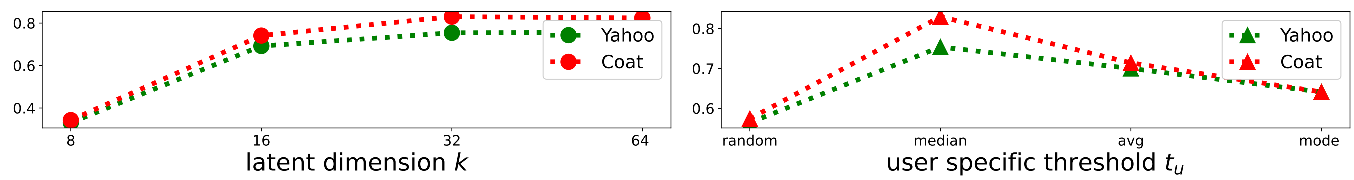



Fig. 7. Paremeter Analysis with NDCG↑.

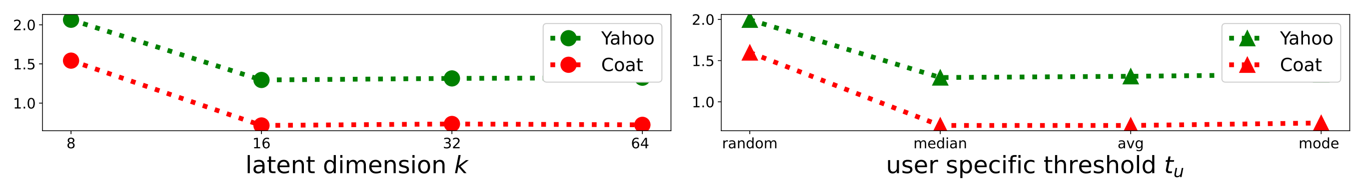



Fig. 8. Paremeter Analysis with MSE↓.

for training VAEs under MNAR have been developed. The not-MIWAE [10], which handles MNAR data by explicitly modeling the conditional distribution of the missing mask, is inspired by MIWAE. The VSAE [28], which uses a selective proposal distribution, effectively learns representations from partially-observed heterogeneous data. An identifiable model for MNAR called GINA is presented by [4]. These methods illustrate the diversity and complexity of the MNAR problem and the challenges of developing effective and efficient models for learning from incomplete data. However, there are still some open issues and limitations (the data limitation and the task limitation we propose in the Introduction) that need to be addressed in future research. Due to the complicated recommender scenarios [29–32], MNAR situation should be taken back to the research spot.

# 6 Concluding Remarks

We present a novel lightweight framework that can tackle missing-not-at-random (MNAR) recommendation tasks without resorting to missing-at-random (MAR) prior propensity, a common assumption in existing MNAR models. Our key insight is that user preference is the underlying prior propensity that governs both MAR and MNAR data, and we leverage this insight to design a unified model that can learn from both types of data. We conduct extensive experiments on public datasets and show that our framework outperforms state-of-the-art MNAR models in various real-world scenarios. Our work bridges the gap between MAR and MNAR data and enables existing MNAR models to benefit from real-world MNAR data without incurring high data collection costs of MAR data.

In future work, we plan to explore the applicability and adaptability of our MNAR-MAR data framework to other domains, especially those that involve data-sensitive, data-expensive tasks, such as computer vision, sequential data analysis, and pre-training model construction. We also aim to investigate our framework's theoretical properties and limitations, such as convergence, robustness, and generalization ability.

# Acknowledgment

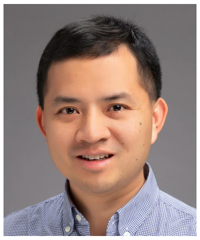
This work is supported by the National Natural Science Foundations of China under Grant No. 62472196, No. 61772230, No.61976102, No.U19A2065, and No. 61972450, Natural Science Foundation of China for Young Scholars No. 62002132. Jilin Education Science Foundation JJKH20221010KJ. Jilin Science and Technology Research Project 20230101067JC. Also supported by the National Natural Science Foundation of China under Grant Nos. 62176014, the Fundamental Research Funds for the Central Universities.

# References

[1] Q. Guo, F. Zhuang, C. Qin, H. Zhu, X. Xie, H. Xiong, and Q. He, "A survey on knowledge graph-based recommender systems," 2020.

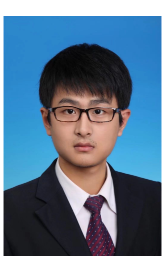
[2] J. Chen, H. Dong, X. Wang, F. Feng, M. Wang, and X. He, "Bias and debias in recommender system: A survey and future directions," 2020.
[3] B. M. Marlin and R. S. Zemel, "Collaborative prediction and ranking with non-random missing data," pp. 5–12, 2009.
[4] C. Ma and C. Zhang, "Identifiable generative models for missing not at random data imputation," 2021.
[5] J. M. Hernández-Lobato, N. Houlsby, and Z. Ghahramani, "Probabilistic matrix factorization with non-random missing data," PMLR, pp. 1512–1520, 2014.
[6] W. Ma and G. H. Chen, "Missing not at random in matrix completion: The effectiveness of estimating missingness probabilities under a low nuclear norm assumption," 2019.
[7] T. Schnabel, A. Swaminathan, A. Singh, N. Chandak, and T. Joachims, "Recommendations as treatments: Debiasing learning and evaluation," PMLR, pp. 1670–1679, 2016.
[8] X. Wang, R. Zhang, Y. Sun, and J. Qi, "Doubly robust joint learning for recommendation on data missing not at random," PMLR, pp. 6638–6647, 2019.
[9] Q. Zhang, L. Cao, C. Shi, and L. Hu, "Tripartite collaborative filtering with observability and selection for debiasing rating estimation on missing-not-at-random data," pp. 4671–4678, 2021.
[10] N. B. Ipsen, P.-A. Mattei, and J. Frellsen, "not-miwae: Deep generative modelling with missing not at random data," 2020.
[11] J. Yoon, J. Jordon, and M. Schaar, "Gain: Missing data imputation using generative adversarial nets," PMLR, pp. 5689–5698, 2018.
[12] S. Ghalebikesabi, R. Cornish, L. J. Kelly, and C. C. Holmes, "Deep generative pattern-set mixture models for nonignorable missingness," 2021.
[13] C. Ma, W. Gong, J. M. Hernández-Lobato, N. Koenigstein, S. Nowozin, and C. Zhang, "Partial vae for hybrid recommender system," 2018.
[14] R. J. Little and D. B. Rubin, "Statistical analysis with missing data," 2019.
[15] D. Liang, L. Charlin, J. McInerney, and D. M. Blei, "Modeling user exposure in recommendation," pp. 951–961, 2016.
[16] W. Wang, F. Feng, X. He, X. Wang, and T.-S. Chua, "Deconfounded recommendation for alleviating bias amplification," 2021.
[17] A. Mnih and R. R. Salakhutdinov, "Probabilistic matrix factorization," pp. 1257–1264, 2008.
[18] Y. Zhang, F. Feng, X. He, T. Wei, C. Song, G. Ling, and Y. Zhang, "Causal intervention for leveraging popularity bias in recommendation," pp. 11–20, 2021.
[19] J. Chen, H. Dong, Y. Qiu, X. He, X. Xin, L. Chen, G. Lin, and K. Yang, "Autodebias: Learning to debias for recommendation," in *Proceedings of the 44th International ACM SIGIR Conference on Research and Development in Information Retrieval*, New York, NY, USA, 2021, p. 21–30.
[20] H. Steck, "Training and testing of recommender systems on data missing not at random," pp. 713–722, 2010.
[21] J. Yi, J. Lee, S. J. Hwang, and E. Yang, "Sparsity normalization: Stabilizing the expected outputs of deep networks," 2019.
[22] A. Sportisse, C. Boyer, and J. Josses, "Estimation and imputation in probabilistic principal component analysis with missing not at random data," 2020.
[23] A. Sportisse, C. Boyer, and J. Josse, "Imputation and low-rank estimation with missing not at random data," *Statistics and Computing*, vol. 30, no. 6, pp. 1629–1643, 2020.
[24] J. Chen, H. Dong, Y. Qiu, X. He, X. Xin, L. Chen, G. Lin, and K. Yang, "Autodebias: Learning to debias for recommendation," in *Proceedings of the 44th International ACM SIGIR Conference on Research and Development in Information Retrieval*, New York, NY, USA, p. 21–30.
[25] M. Morik, A. Singh, J. Hong, and T. Joachims, "Controlling fairness and bias in dynamic learning-to-rank," pp. 429–438, 2020.
[26] X. Wang, R. Zhang, Y. Sun, and J. Qi, "Combating selection biases in recommender systems with a few unbiased ratings," pp. 427–435, 2021.
[27] Z. Wang, X. Chen, R. Wen, S.-L. Huang, E. Kuruoglu, and Y. Zheng, "Information theoretic counterfactual learning from missing-not-at-random feedback," *Advances in Neural Information Processing Systems*, vol. 33, pp. 1854–1864, 2020.
[28] Y. Gong, H. Hajimirsadeghi, J. He, T. Durand, and G. Mori, "Variational selective autoencoder: Learning from partially-observed heterogeneous data," pp. 2377–2385, 2021.
[29] Y. Xu, Y. Yang, J. Han, E. Wang, F. Zhuang, J. Yang, and H. Xiong, "Neuo: Exploiting the sentimental bias between ratings and reviews with neural networks," *Neural Networks*, vol. 111, pp. 77–88, 2019.
[30] E. Wang, M. Zhang, Y. Xu, H. Xiong, and Y. Yang, "Spatiotemporal fracture data inference in sparse urban crowdsensing," in *IEEE INFOCOM 2022 - IEEE Conference on Computer Communications, London, United Kingdom, May 2-5, 2022*. IEEE, 2022, pp. 1499–1508.
[31] Y. Xu, T. Li, Y. Yang, W. Chen, and L. Yue, "An adaptive category-aware recommender based on dual knowledge graphs," *Inf. Process. Manag.*, vol. 61, no. 2, p. 103636, 2024.
[32] Y. Jiang, Y. Yang, Y. Xu, and E. Wang, "Spatial-temporal interval aware individual future trajectory prediction," *IEEE Trans. Knowl. Data Eng.*, vol. 36, no. 10, pp. 5374–5387, 2024.

**Yuanbo Xu** received his B.E., M.E., and Ph.D. in computer science and technology from Jilin University, Changchun, in 2012, 2015, and 2019. He is an associate professor and Ph.D. supervisor in the College of Computer Science and Technology at Jilin University, Changchun. He is also a visiting scholar in the Management Science and Information Systems Department at Rutgers, the State University of New Jersey. His research interests include data mining applications, recommender systems, and mobile computing. He has published 40+ research results in journals such as IEEE TKDE, IEEE TMC, IEEE TMM, IEEE TNNLS, ACM TKDD, and conferences such as ICDE, INFOCOM, CIKM, and ICDM.



**Fuzhen Zhuang** received the B.E. and Ph.D. degrees in computer science from the Institute of Computing Technology, Chinese Academy of Sciences, Beijing, China, in 2006 and 2011, respectively. He is currently a full Professor with the Institute of Artificial Intelligence, Beihang University. He has published more than 100 papers in some prestigious refereed journals and conference proceedings such as the IEEE TRANSACTIONS ON KNOWLEDGE AND DATA ENGINEERING, the IEEE TRANSACTIONS ON CYBERNETICS, the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, the ACM Transactions on Knowledge Discovery from Data, the ACM Transactions on Intelligent Systems and Technology, Information Sciences, Neural Networks, SIGKDD, IJCAI, AAAI, TheWebConf, SIGIR, ICDE, ACM CIKM, ACM WSDM, SIAM SDM, and IEEE ICDM. His research interests include transfer learning, machine learning, data mining, multitask learning, knowledge graph, and recommendation systems. He is a Senior Member of CCF. He was a recipient of the Distinguished Dissertation Award of CAAI in 2013.

**En Wang** received his B.E. degree in software engineering from Jilin University, Changchun, in 2011, his M.E. degree in computer science and technology from Jilin University, Changchun, in 2013, and his Ph.D. in computer science and technology from Jilin University, Changchun, in 2016. He is currently an Associate Professor in the Department of Computer Science and Technology at Jilin University, Changchun. He is also a visiting scholar in the Department of Computer and Information Sciences at Temple University in Philadelphia. His current research focuses on the efficient utilization of network resources, scheduling and drop strategy in terms of buffer-management, energy-efficient communication between human-carried devices, and mobile crowdsensing.



**Chaozhuo Li** received his Ph.D. degree in computer software and theory from School of Computer Science and Engineering, Beihang University, Beijing, China, in 2020. He is currently an associate professor at the Key Laboratory of Trustworthy Distributed Computing and Service (MoE), Beijing University of Posts and Telecommunications. He has published over 20 papers, such as NeurIPS, AAAI, SIGIR, ICDM and CIKM. His research interests include data mining and social network analysis.

**Jie Wu** is the Director of the Center for Networked Computing and Laura H.Carnell professor at Temple University. He also serves as the Director of International Affairs at College of Science and Technology. He served as Chair of Department of Computer and Information Sciences from the summer of 2009 to the summer of 2016 and Associate Vice Provost for International Affairs from the fall of 2015 to the summer of 2017. Prior to joining Temple University, he was a program director at the National Science Foundation and was a distinguished professor at Florida Atlantic University. His current research interests include mobile computing and wireless networks, routing protocols, cloud and green computing, network trust and security, and social network applications. Dr. Wu regularly publishes in scholarly journals, conference proceedings, and books. He serves on several editorial boards, including IEEE Transactions on Mobile Computing, IEEE Transactions on Service Computing, Journal of Parallel and Distributed Computing, and Journal of Computer Science and Technology. Dr. Wu was general co-chair for IEEE MASS 2006, IEEE IPDPS 2008, IEEE ICDCS 2013, ACM MobiHoc 2014, ICPP 2016, and IEEE CNS 2016, as well as program co-chair for IEEE INFOCOM 2011 and CCF CNCC 2013. He was an IEEE Computer Society Distinguished Visitor, ACM Distinguished Speaker, and chair of the IEEE Technical Committee on Distributed Processing (TCDP). Dr. Wu is a CCF Distinguished Speaker and a Fellow of the IEEE. He is the recipient of the 2011 China Computer Federation (CCF) Overseas Outstanding Achievement Award.